following coefficient tolerances: A, 70-120%; E, 50-500%; b', 100-500%; T, 70-120%.

Note that there is great latitude in estimating the flight coefficients E and b'. This flexibility can be used to design a slower autopilot by biasing the estimates upwards. It is recommended from simulation that, during the last quarter second, the commands n_c should be set to zero, which allows the angle of attack to diminish before intercept in order to present a more favorable missile attitude for warhead performance. Actually, the missile attitude at intercept can be set to optimize warhead effectiveness.

Conclusions

Optimal control theory is applied to the linear autopilot pitch plane design of a skid-to-turn missile with consideration for implementation. A closed-form analytic expression of the optimal control law is derived. The autopilot gains are adapted to the environment model reference and varied as functions of time-to-go to minimize the performance index. The path constraint is applied thoughout the whole terminal homing period and infinite penalty is imposed at intercept. The following can result from application of the optimal control law: improvement in miss distance and pitch acceleration history, good immunity to guidance noise, tolerance for radome error, compatibility with short handover times, bringing of autopilot states to rest at intercept, and reduction of the angle of attack at intercept.

Acknowledgment

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Reliability Considerations in the Placement of Control System Components

Raymond C. Montgomery*
NASA Langley Research Center, Hampton, Virginia
and
Wallace E. Vander Velde†
Massachusetts Institute of Technology
Cambridge, Massachusetts

Introduction

FUTURE space missions may involve very large and highly flexible spacecraft. Reference 1 considers a microwave radiometer 100 m in diameter. In that example, many actuators and sensors were needed to meet radiometry requirements. Also, to amortize the mission costs, mission life is ideally 20 years or more with occasional revisits for repair and replacement. One mission, the manned space station, is projected to operate indefinitely. These long-life requirements, large numbers of sensors and actuators, and heavy dependence on the proper operation of the control system call for the design ground rule that components may fail during the mission. Hence, components must be placed effectively on the structure and on-line automatic failure detection, identification, and control system reconfiguration (FDI&R) capability is needed. Unfortunately, this capability at the overall system level is still in the research stage. Until now, advanced redundancy management techniques have been used only in research programs. For example, the NASA F8-DFBW program demonstrated analytic redundancy management of sensors in a high-performance research aircraft.[2] In that activity, the problem was simpler than the current one since flexibility was not an overbearing issue.

The problem of selecting component locations is addressed in this paper. The goal is to provide the designer a method of evaluating different sets of locations with regard to their performance as it affects component reliability. This was done in Ref. 3 for the problem of static shape control of a beam. In that paper, optimal locations of force actuators on a beam were found to bend the beam to fit a prescribed parabolic shape. It was shown that reliability should have substantial impact on the selection of the locations of the actuators. This concept has been applied to static shape control of a grid in Ref. 4. Therein the criterion was shown to favor clustering the force actuators for long-life missions, since any one may substitute for the others. For short-life missions (or missions where reservicing can be conducted frequently), physically distributing the actuators is favored.

This Note extends the approach of Ref. 4 to dynamic vibration suppression and applies the theory developed to a grid. Reference 5 treats the problem by constructing a measure of controllability and suggests optimizing the average value of that measure taken over the design mission life. Herein, vibration suppression is treated using optimal regulator theory. First, the general discrete regulator theory is overviewed. Then, the reliability of the components is considered and the failure characteristics of the components are modeled. Finally, results of the analysis are presented that compare the performance of actuator location sets on a grid. Optimality is shown to depend on the mission life or reservicing duration interval.

Basic Analytic Considerations

Engineering practice in the modeling of large space structure dynamics has centered around the use of finite element modeling. Herein, such a model is assumed given and valid for the purpose of control system design—the main interest here

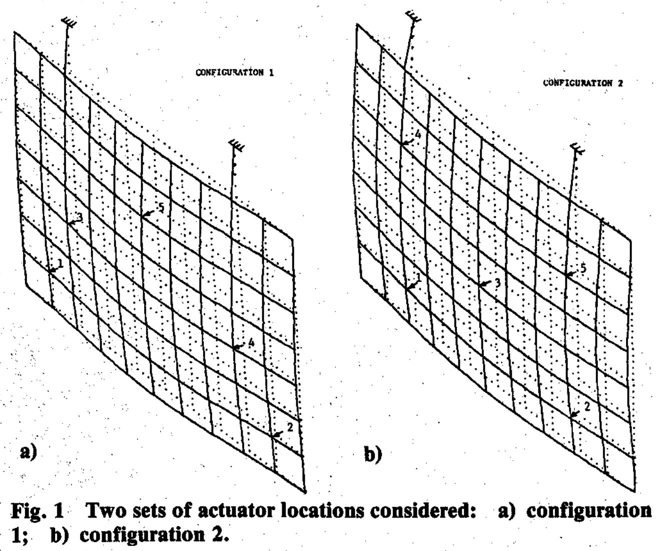

Fig. 1 Two sets of actuator locations considered: a) configuration 1; b) configuration 2.

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*Aerospace Technologist, Spacecraft Control Branch. Member AIAA.

†Professor of Aeronautics and Astronautics, Department of Aeronautics and Astronautics. Fellow AIAA.

being treatment of the reliability issue. The model is of the form

$$\dot{x} = Ax + Bu \quad (1)$$

where x is an n-dimensional vector consisting of pairs of modal amplitude/velocity elements and u an m-dimensional input vector consisting of the forces or moments applied to the structure. The structure used in this example is the NASA Langley grid experiment test structure described in Ref. 6. The experimental apparatus uses a digital computer to generate torque and moment commands to actuators placed on the grid; hence, the discrete form of model is most appropriate,

$$x_{k+1} = \phi x_k + \Gamma u_k$$

Given the analytic model, a measure of performance for vibration suppression is

$$J = \int_0^T (x'Qx + u'Ru)\mathrm{d}t$$

with matrices $Q = Q' > 0$ and $R = R' > 0$. This performance equation can be written in a form convenient to use of the discrete model,

$$J = \sum_{k=0}^{N} (x_k'Qx_k + x_k'Wu_k + u_k'Ru_k)$$

The term involving W can be eliminated using the control variable transformation $u_k = -Fx_k + v_k$ and appropriately selecting the matrix F. This results in the model equation

$$x_{k=1} = \phi x_k + \Gamma v_k \quad (2)$$

together with the performance measure equation

$$J = \sum_{k=0}^{N} (x_k'Qx_k + v_k'Rv_k) \quad (3)$$

The discrete optimal linear regulator problem is that of determining the control sequence v_k that minimizes J. In our application, the only element of concern is the resulting optimal performance measure J^*, which is of the form

$$J^* = x_0'P_0x_0 \quad (4)$$

where x_0 is the initial value of the state x and P_0 the solution of the discrete Ricatti equation at stage 0. This last equation provides a measure of the best realizable performance of a control system, assuming that a certain number of the actuators are functional. Computational algorithms for constructing the coefficient matrices needed in Eqs. (2) and (3) and for solving for P_0 in Eq. (4) can be found in the ORACLS computer-aided design package.[7]

Incorporating Reliability

If one does not consider possible failure, the placement of control system components that produces the best system responses is the set of locations which minimizes the function J^*. Reliability becomes an issue when the components of the control system are used for durations approaching their mean time to failure, t_m. In order to obtain a methodology for selecting the best locations for control system components, it is necessary to specify the class of failures to be considered and the action to be taken in response to the failure. Here, it is assumed that the failure is detected whenever it occurs and that the control actuator or actuators involved produce a zero force or moment output. Also, it is assumed that the system will use the remaining working actuators in an optimal manner so that Eq. (4) still describes the optimal system performance index. Hence, for any failure case, one may compute the optimal realizable performance of the control system by constructing the appropriate B matrix of Eq. (1) and performing the calculations leading to the P_0 matrix of Eq. (4).

To obtain an analytic model, a finite element analysis of the grid was performed that included the suspension cables. Figure 1 shows the grid structure; the arrows indicate the two sets of actuator locations considered. Nodes were placed at each overlapping joint on the grid, the ceiling attachment points of the cable, and every ½ ft along the cable. The grid elements connecting the nodes were modeled as bending elements and the cable elements as two-force members. Thus, a total of 165 elements were included in the model. Four degrees of freedom appropriate for motion normal to the plane of the grid were considered. No damping was included in the model. Thirty modes were obtained from this analysis.

The best realizable performance of the control system, taken over a 5 s time interval with a sampling time interval of 1/32 s, is shown in Table 1. The Q and R matrices are identity matrices for this example. The table lists the failure modes considered. The performance function was based on initial conditions of unity in the velocity of each modal coordinate. It shows that under some failure modes configuration 1 is better than 2, whereas for others the opposite is true.

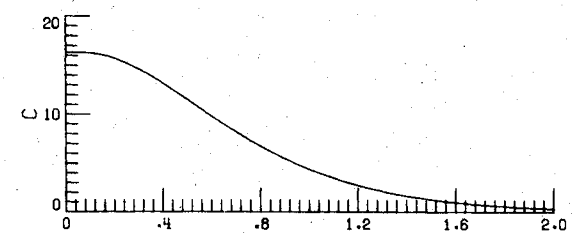

Fig. 2 Variation of the weighted cost with T/T_m for configuration 1.

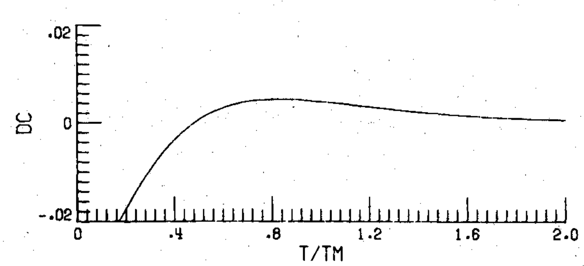

Fig. 3 Increment in weighted cost of configuration 2 over that of configuration 1 as a function of T/T_m.

Table 1 Summary of the best performance achievable for the failure modes considered in the design set

Mode	Actuator state[a]	Optimal costs	
		Configuration 1	Configuration 2
1	00000	16.305	16.274
2	10000	16.557	16.519
3	01000	16.555	16.518
4	00100	16.401	16.324
5	00010	16.402	16.423
6	00001	16.435	16.424
7	11000	16.533	16.582
8	10100	16.704	16.805
9	10010	16.703	16.683
10	10001	16.657	16.682
11	01100	16.705	16.806
12	01010	16.705	16.684
13	01001	16.659	16.683
14	00110	16.902	16.919
15	00101	16.837	16.919
16	00011	16.837	16.790

[a] 0 = functioning component, 1 = a failed component.

The control system designer has the problem of selecting a configuration using the information in Table 1. One approach to this problem is to construct another function from the table that indicates how the system will most probably perform during a mission. This requires modeling the failure characteristics of the control system components as a function of mission time t. For this example, the failure characteristics are assumed to be identical for each component and described by an exponential probability density function for the time of failure as a function of mission time,

$$p(t) = \exp(-t/t_m)/t_m$$

It is suggested that the designer should weight J^* of Table 1 according to the probability that the mode will occur at the end of the mission. This has been done and Fig. 2 shows the variation of this weighted performance indicator as a function of t/t_M. Note that at $t/t_M = 0$, the weighted cost function is equal to that in Table 1 for the mode with no failures. Also, note that the weighted cost goes to zero as t/t_M increases. This occurs because the set of failure modes considered by the designer is not complete. It is possible that three, four, or even five simultaneous failures may occur. These are not considered in the design set so as not to bias the selection of actuator configuration with catastrophic failure modes. Figure 3 is a graph of the incremental cost of configuration 2 over configuration 1. The notable point is that configuration 2 is preferable to 1 for short missions, whereas configuration 1 is preferable to 2 for long missions.

Conclusions

This Note has described a methodology that allows the control system designer to select from among a set of possible actuator locations the one that is best for vibration suppression considering the reliability of the components. This extends the results of earlier work that was developed using static modeling for the shape control problem to problems that involve the dynamics of a flexible spacecraft. The method has been applied to a grid structure as an example. In that case, it was shown that optimal locations depend on the design mission life. The method generally involves, for each candidate location set, determining the best achievable performance of the control system for all failure modes that the designer wishes to consider. These values of performance are then used to construct the criterion function by taking the probability-weighted sum of the performance measures for each failure mode. The actuator locations are then chosen to minimize this cost criterion.

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Subaliases in the Frequency Response of Digitally Controlled Systems

Hsi-Han Yeh*
University of Kentucky, Lexington, Kentucky

I. Introduction

IN a recent paper, Whitbeck, Didaleusky, and Hofmann[1] extended the concept of the traditional "sampled-spectrum" frequency response for discretely excited continuous systems. When a sinusoidal wave is input to a discretely excited continuous system, N sine waves at different alias frequencies are required to match the continuous steady-state time response at the sampling instants and at $N-1$ equally spaced intersample points. In the special case of $N=1$, this reduces to the traditional concept of the sampled-spectrum frequency response for sampled-data systems. Letting N approach infinity gives an infinite spectrum for the continuous steady-state response of a discretely excited continuous system. This theory enables one to write an exact expression of the time response sampled at any rate that is an integer multiple of the sampling rate of the system. The practical value of knowing such an expression of the output is evident.

However, in the derivation of Whitbeck, Didaleusky, and Hofmann,[1] only positive aliases of the input frequency are included in the representation of the sampled continuous output. This result is correct for the case where N is finite, but contains only half of the spectral components necessary for an asymptotic representation of the continuous output for the limit case where N approaches infinity. In this paper, a new derivation, which includes subaliases in the spectral representation of the sampled output, will be presented. It will be shown that, as the output sampling rate N approaches infinity, the infinite spectrum of the continuous output contains all aliases and subaliases of the input frequency. Also presented will be a direct derivation of the infinite spectrum of the continuous output, without invoking the expression of the sampled steady-state response. This confirms the correct representation of the continuous steady-state output of discretely excited continuous systems in response to a single sinusoidal input. This derivation brings forth a unified concept of frequency response, which enables one to write the spectral representation of the output of a discretely excited continuous system on the basis of the frequency response of the continuous system.

II. Frequency Components in the Sampled Output

Consider the system of Fig. 1, where $G(s)$ represents an arbitrary transfer function and $M(s)$ represents an arbitrary data hold. Let the input be a unit amplitude exponential ϵ^{jbt} and the output be sampled with period T/N. Using multirate sampling results (see Appendix of Ref. 1) yields

$$C^{T/N} = [GMR^T]^{T/N} = (GM)^{T/N} R^T$$

$$= (GM)^{T/N} \frac{z^N}{z^N - \epsilon^{jbT}} \qquad (z \triangleq \epsilon^{sT/N}) \tag{1}$$

where the notation follows Ref. 1 and the superscript denotes the period of sampling operation. In the time domain, a sampled function $[r(t)]^T$ or r^T is defined by

$$[r(t)]^T \triangleq \sum_{n=-\infty}^{\infty} r(n)\delta(t-nT) = r(t) \sum_{n=-\infty}^{\infty} \delta(t-nT) \tag{2}$$

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*Associate Professor, Department of Electrical Engineering.